22. Assembly & Mobility Analysis of an Optical Mount *[partial solutions]*

1. Explain how the two main parts are held together.

The two main bodies are held together thanks to two springs in tension mounted in the arms, between each adjuster and the ball. Apart from these springs, there are no fixed links between the two main bodies, but only spherical contact points as shown in the figure.

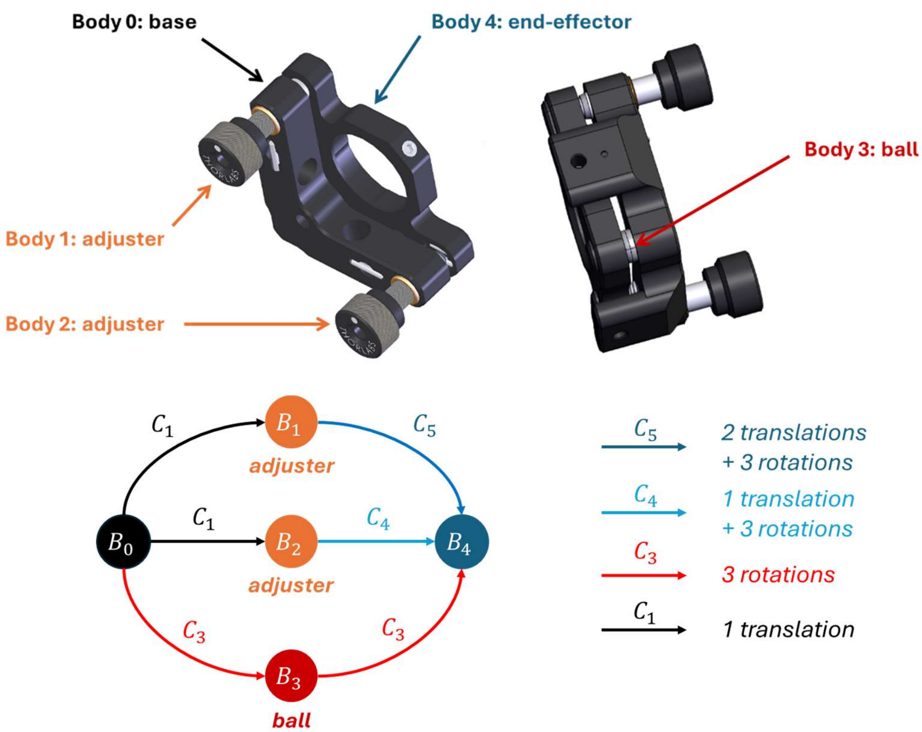
2. Draw the graph of this mechanism and do the mobility analysis (a) considering the ball as an independent body, (b) considering the ball as part of the base body.

Method 1. We consider the ball as an independent body.

The graph is represented on the figure below. The mobility analysis gives:

$$M = \cdots = 5$$

- There are _____ internal degrees of freedom: Body 3 can rotate along the three rotation axes without causing any motion of Body 4 with respect to Body 0.
- Hence, the mechanism has _____ "productive" degrees of freedom (_____ per adjuster).



Top graph of the mechanism with the links between bodies indicated.

Method 2. We consider the ball as attached to one of the two main bodies (base or end-effector).

Let us imagine that the sphere is attached to the base (for instance glued, so that no motion is possible). We can perform a similar mobility analysis. This time, Body 3 disappears on the graph and becomes part of Body 0. The new analysis gives:

$$M = \cdots = 2$$

We see that indeed, as expected, only two degrees of freedom remain.

3. Locate the position of the center of rotation.

From a rapid observation of the mechanism, we find that the center of rotation is located at the center of the ball (Body 3 in the previous figure).

4. Assume that the distance between the sphere on the lower left corner and the two other spheres is the same and is called L. Additionally, assume that the point M is in the middle of the optical mount at a distance $L/2$ away of the two orthogonal axes formed by the three spheres.
   a. What is the trajectory of point M?

From what precedes, the point M does an arc motion.

   b. Is the mirror doing perfect rotations? If not, express the amplitudes of the parasitic motions ε_x and ε_y as a function of a small motion of the adjusters δ and L.

The mirror is therefore not doing a perfect rotation. For small motion of the adjusters, say δ, the amplitude of the parasitic motion (ε) will be exactly:

$$\tan(\alpha) = \frac{\delta}{L}$$

$$\varepsilon_x = \frac{L}{2}\tan(\alpha) = \frac{\delta}{2}$$

$$\varepsilon_y = \frac{\delta}{2}\tan(\alpha) = \frac{\delta^2}{2L}$$

This motion is therefore typically __________ for small excursions of the adjusters.

   c. How could we get rid of any parasitic motions?

In certain cases, these parasitic motions are still not acceptable and other more advanced designs are necessary. Optical mounts with center of rotation positioned on the mirror are sometimes called “true gimbals mounts” and typically allow perfect rotations around the mirror.

23. The Kinematics Couplers *[full solutions]*

(Adapted from the final exam 2019)

23.1. Couplers Analysis

1. Using graph theory and mobility analysis with CGK formula, show that both systems are kinematically equivalent. We expect a graph for each system, and calculation of their mobility.

Two straightforward graphs can be drawn, each of them having two bodies only ($N_B = 2$) and three links ($N_L = 3$). The nature of these links is not the same for a Maxwell or a Kelvin coupling.

There are different ways of analyzing the kinematics:

i. The first approach is to analyze each leg as a different joint. This analysis corresponds to the graphs described just above, where they show only three links.
ii. Another solution is to go down to the minimal joint, which is a sphere to a plane type of joint. If you follow such analysis, the graphs shall show six links and not three.

We use the CGK formula:

$$M = \sum_{i=1}^{5} n_i \cdot C_i - m \cdot \left(N_{L,g} - N_{B,g} + 1\right)$$

	Maxwell	**Kelvin**
Method (i)	Three identical C_4 joints (sphere in a v-groove), two bodies, three links: $M = (3 \cdot 4) - 6 \cdot (3 - 2 + 1)$ $= 12 - 12 = 0$	A C_3 joint (pivot), a C_4 joint (sphere in a v-groove) and a C_5 joint (sphere on a plane), two bodies, three links: $M = (3 + 4 + 5) - 6 \cdot (3 - 2 + 1)$ $= 12 - 12 = 0$
Method (ii)	C_5 joints (sphere on a plane) only, two in each of the three legs, two bodies, six links: $M = (2 \cdot 3 \cdot 5) - 6 \cdot (6 - 2 + 1)$ $= 30 - 30 = 0$	C_5 joints (sphere on a plane) only: three in the first leg, two in the second and one in the third, two bodies, six links: $M = ((3 + 2 + 1) \cdot 5) - 6 \cdot (6 - 2 + 1)$ $= 30 - 30 = 0$

2. Discuss the general function of these devices and their specific purposes.

These devices are used to precisely position two bodies, so that there exists only one stable assembly coupling the parts. Such system is isostatic and uniquely defined without being over-constrained.

3. Assume that instead of the triangular shape for the detachable element (as shown in **Figure 23**), a simple cylindrical plate is now used and contains the spherical contacts (as shown below). In such case, can a vertical load (i.e. oriented perpendicular to the plane of the mechanism) be applied at any point on the disk? Comment on it.

A proper answer would be to draw a triangle with the spheres as vertices and to understand that the structure is mechanically stable if and only if the vertical force is applied at a point located within the triangle (see lecture notes).

4. Discuss the important physical parameters (i.e. intrinsic properties of the materials) to optimize when selecting the materials for:
    a. The detachable plate to minimize deflection and maximize structural stability
    Young's modulus

    b. The surfaces in direct contact
    Hardness

    c. Maximizing the accuracy of the system while in operation under varying temperature
    Coefficient of thermal expansion

    d. Heavy load operation while being lightweight
    Optimizing the ratio between Young's modulus and density

5. What advantage / disadvantage do you see for each mechanism when comparing them?

Depending on fabrication tolerances, the Kelvin coupling defines more precisely one point at the center of the leg having a C_3 joint. The Maxwell coupling has uncertainties in position depending on tolerances between the two parts, as opposed to only one like for the Kelvin case. The exact position after machining will not be uniquely defined from design.

Kelvin couplings are more expensive to fabricate. The load is also not uniformly distributed. If a load P is applied, one surface receives $P/3$, while the other receive $P/6$ or $P/9$. The choice of material should be done carefully for the singular contact point in the case of the Kelvin coupling.

23.2. Reverse Engineering

6. To what type of coupling is this device equivalent to?
    It is equivalent to a Maxwell coupling.

7. What manufacturing operations have been used for the main parts (in black)? Discuss a credible fabrication process sequence as well as a possible assembly sequence, and the choices that were made (in particular regarding the pins orientation).

The answer is conventional machining:
- Drilling operations are made for the holes (as well as the spherical cavities) and some end-milling for the contours (chamfers) and the center hole.
- Tapping is used for the threads.

Parts are assembled:
- Most likely glued for the metallic balls and magnets.
- For the "v-groove" equivalent made with steel pins, they were most likely gently force-fitted in the holes on the sides.

A credible assembly sequence is to start by inserting the pins and the spheres, and then the magnets. The reverse sequence might be possible, but there is a risk to see some parts sticking to the magnets that are quite strong (if the pins if mishandled in particular).

Note that even if the 90° orientation between the pins (as opposed to the standard 45° seen earlier in the exercise) does not change dramatically the function of the device, it simplifies the machining process as only 90°-rotating operations can be done. In addition, the outer surface can be used for referencing.

24. Soldering Processes *[full solutions]*

1. What is the melting temperature of this solder?

$$T_m = 183°\text{C}$$

2. Suppose we are using this solder to form a connection between a pad on a PCB and a pad on an integrated circuit. Assume that we are using this connection in an audio amplifier that is intended to drive an $8\ \Omega$ loudspeaker at $100\ \text{W}$. What is the current flowing through the solder?

$$P = I^2R, \qquad I = 3.54\ \text{A}$$

3. Assume both pads are $50\ \mu\text{m}$ squares, and that the thickness of the connection is also $50\ \mu\text{m}$ (in other words, assume the solder forms a cube that is $50\ \mu\text{m}$ on each side). Ignoring any heat losses through the pads, what is the maximum amount of time that this current can flow through the pads without the solder re-melting during operation?
Hint. You may find the following information useful:
   - *Electrical resistivity of Sn-37Pb:* $0.153\ \mu\Omega \cdot \text{m}$
   - *Specific heat capacity of Sn-37Pb:* $0.21\ \text{J} \cdot \text{g}^{-1} \cdot °\text{C}^{-1}$
   - *Density of Sn-37Pb:* $8.4\ \text{g} \cdot \text{cm}^{-3}$
   - *Assume we start at* $25°\text{C}$
   - *Ignore the difference between liquidus and solidus, i.e., just use the lower bound*

We first compute the solder's resistance:

$$R = 0.153\ [\mu\Omega \cdot \text{m}] \cdot \frac{50\ [\mu\text{m}]}{50\ [\mu\text{m}] \cdot 50\ [\mu\text{m}]} = 3.06\ \text{m}\Omega$$

We can now estimate the power delivered to the solder:

$$P = (3.55\ [\text{A}])^2 \cdot 3.06\ [\text{m}\Omega] = 38.3\ \text{mW}$$

Hence, the heat to reach 183°C is:

$$E = 0.21\ [\text{J} \cdot \text{g}^{-1} \cdot °\text{C}^{-1}] \cdot (183°\text{C} - 25°\text{C}) \cdot 8.4\ [\text{g} \cdot \text{cm}^{-3}] \cdot (50\ [\mu\text{m}])^3 = 34.8\ \mu\text{J}$$

And finally, the time to melt is given by:

$$t = \frac{E}{P} \cong 0.909\ \text{ms}$$

4. Clearly, this is an oversimplification, and in fact, the time to melt is much longer in reality. Why?

There is significant heat loss, particularly through the pads. We should therefore actually solve this problem by accounting for the heat loss through conduction from the pads, etc.

5. If we wanted to include the effect of heat loss, what equations would we include in our model to determine the relationship between current flowing through the connection and the onset of melting? Assume steady state conditions.

Power delivered to solder is I^2R, where R is calculated as above.

Heat loss rate through each pad is $k \cdot A \cdot \Delta T/d$, where k is the pad thermal conductivity, A is the pad area, ΔT is the temperature difference across the pad, and d is the pad thickness.

We could form a very simple equation framework by assuming the temperature is uniform across the solder at just below the melting point, i.e., $183°\mathrm{C}$, and could assume that the outside of the pads ware at room temperature. Using the thermal conductivity of the pads, we could find the heat loss. At steady state, this would give us an indication of the maximum current, since the heat loss must exactly balance the power in.

For a more accurate system, we could actually solve the differential equation to model heating up from room temperature.

6. Even when we use more accurate analysis, it turns out that Sn-37Pb is not good for such high current applications, and we are therefore forced to use solders capable of handling higher temperature. Suppose we were to use a solder with a higher percentage of Pb (e.g., 60% Pb solder). At 185°C (i.e., just above the eutectic line), what phase(s) is (are) present and at what compositions?

The point corresponding to 40 wt% Sn and $185°\mathrm{C}$ on the diagram lies in an alpha + liquid region. Therefore, both the alpha phase and the liquid phase are present.

Extend a horizontal line from this point to the closest phase boundaries. Drop a vertical line down from the phase boundary between the alpha + liquid two phase region and the liquid region to find the composition of the liquid phase. Drop a vertical line down from the other intersection to find the composition of the alpha phase. In this case the composition of the liquid phase is 61.9 wt% Sn and the composition of the alpha phase is 19.2 wt% Sn.

7. What is the relative amount of each phase present, in mass fraction?

This step requires the use of the lever rule since the alloy consists of two phases:

$$W_\alpha = \frac{C_l - C_s}{C_l - C_\alpha} = \frac{61.9 - 40}{61.9 - 19.2} \cong 0.513$$

Therefore, the α phase makes up 51.3% of the alloy and the liquid phase makes up 48.7% of the alloy.

8. What components would you find in the solid upon solidification of the solder to just below the eutectic line, at with what mass fractions?

Dropping the lines from the intersections as before shows that the composition of the alpha phase is 19.2 wt% Sn and the composition of the beta phase is 97.5 wt% Sn. The lever rule is used again:

$$W_\alpha = \frac{C_\beta - C_s}{C_\beta - C_\alpha} = \frac{97.5 - 40}{97.5 - 19.2} \cong 0.734$$

The mass fraction of the α phase is 0.734, or 73.4%, and that of the β phase is 0.266, or 26.6%.

9. Would you expect this solder to have better thermal performance under high current conditions? Give reasons.

Since the solid consists of alpha and beta phase, we would expect better thermal performance. Indeed, both melt above the eutectic temperature, as does the combined form as well.

Consider the use of solder balls to form connections for a modern integrated circuit chip to a package. Assume the pads are $20\ \mu\text{m}$ squares, with $10\ \mu\text{m}$ spacing between the pads. For simplicity, assume that the solder balls are perfect spheres with diameter of $20\ \mu\text{m}$ when the chip is placed in contact with the package. Therefore, at this initial point of contact, the separation between the plane of the chip pad and the package pad is exactly $20\ \mu\text{m}$. For all the questions below, use 2D analysis (i.e. there is no need to do 3D calculations). You may find the figure in Slide 50 of the packaging course notes useful.

10. Pressure is applied to the chip to form a good bond, which results in the solder balls being squeezed into ellipses. Assuming area is conserved, what is the die-to-package spacing at which adjacent pads are short circuited?

Area of spherical ball is calculated to be $\mathcal{A} = 314\ \mu\text{m}^2$. At the short circuit condition, each ball will have distorted into an ellipse that just touches the nearest neighbour (i.e. the lateral diameter will be $30\ \mu\text{m}$).

By conservation of area, the vertical diameter will be approximately $13.3\ \mu\text{m}$ (i.e. the die and package will have been squeezed by about $6.7\ \mu\text{m}$).

11. Obviously, we don't want this to happen. Assume the minimum allowable spacing between the ellipses in $5\ \mu\text{m}$. What is the die-to-package spacing under this condition?

With an ellipse spacing of $5\ \mu\text{m}$, the lateral diameter is $25\ \mu\text{m}$. By conservation of area, the vertical diameter will be approximately $8\ \mu\text{m}$ (i.e., the die and package will have been squeezed by about $2\ \mu\text{m}$).

12. Assume the package is perfectly flat, but the chip has some curvature such that the outermost pads are higher than the center pad. What is the maximum height difference before we obtain open circuits at the edges?

Since the squeeze is $2\ \mu\text{m}$, the height difference cannot exceed this amount before we start obtaining open circuits at the edges.